

Mathematical Model of E-learning System Based on Fuzzy Logic

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ABSTRACT One of the key challenges of modern e-learning is the timely identification of students at risk of expulsion in conditions of uncertainty and incompleteness of input information. The purpose of this work is to develop a mathematical model of e-learning system based on fuzzy logic that can predict the probability of successful completion of the course.

The proposed model considers key factors of academic success, such as activity, time spent in the system, average score, attendance, participation in discussions, and test scores. To process input data, triangular belonging functions and a fuzzy rule base are used to formalize uncertainty in educational data. The centroid method is used as a defuzzification method.

The paper formulates and solves the problem of predicting the expulsion of students using an adaptive neuron-fuzzy system. A numerical experiment was conducted on the data of 1000 students, during which the classification accuracy of 81.7% and the value of AUC = 0.90 were achieved, which confirms the high efficiency of the model.

The practical significance of the proposed approach lies in the possibility of its integration into existing e-learning systems for the early identification of students at high risk of academic failure and subsequent adaptive management of the educational process aimed at reducing expulsions and increasing the effectiveness of e-learning.

KEYWORDS e-learning system, fuzzy logic, intelligent educational systems, modeling of student success.

I. INTRODUCTION

Modern e-learning systems are complex cybernetic structures that combine automated algorithms and the human factor. One of the main tasks of such systems is to predict the academic performance of students, which allows timely identification of those who are at risk and adapt the educational process to support them [1].

Traditional assessment methods, such as analysis of GPA, attendance, and activity on the platform, are not effective enough, as the learning process is characterized by significant uncertainty due to subjective factors: student motivation, individual cognitive characteristics, and external conditions [2, 3].

In conditions of high uncertainty, an effective approach is the application of fuzzy logic and neuro-fuzzy systems. Fuzzy sets and fuzzy inference rules allow formalizing uncertainty associated with the educational process and increase the adaptability of the system [4-7]. Thus, work [4] has shown that ANFIS allows taking into account both quantitative and

qualitative aspects of data, which increases the accuracy of predicting academic performance. Additionally, the use of deep neural networks allows modeling complex patterns in student data [8, 9].

Based on the developed mathematical model of the e-learning system, it is planned to formulate an optimization problem to predict the probability of expulsion of students at the end of the semester. Optimization methods based on fuzzy neural networks allow minimizing prediction errors and suggesting effective measures to improve success [10, 11].

II. OVERVIEW OF THE PROBLEM

Predicting the academic performance of students in e-learning systems is an urgent task that is actively developing in the field of educational analytics and information technology. Various methods, from classical machine learning algorithms to deep neural networks and neuro-fuzzy systems, can increase the accuracy of predictions and the adaptability of the learning process.

Study [12] demonstrated the application of Random Forest and XGBoost algorithms to predict student performance, where the models showed high prediction accuracy. However, the authors noted that these methods do not take into account the uncertainty and fuzziness of students' behavior, which limits their adaptability. Similar results were obtained in study [13], where Logistic Regression, SVM, and Random Forest were compared, noting that Random Forest achieved the highest accuracy (AUC = 73%), but did not take into account the fuzziness of the data [14, 15].

Other studies [1, 16] highlighted the effectiveness of machine learning techniques for analyzing large amounts of data, in particular for processing attendance, platform activity, and scores. In particular, study [17] proposed a method of parallel search for information objects in a single information space, which allows for effective processing of large educational data and increases the performance of forecasting systems in e-learning. However, they do not provide adaptability to changes in students' motivation or their cognitive characteristics [18].

Neural networks allow us to model complex patterns in training data and individualize the learning process [19]. For example, the model in paper [20] uses deep neural networks to predict student performance in online courses. It has shown high accuracy, especially in the early stages of learning, but requires significant amounts of data for training and does not provide interpretation of the results, which makes practical use difficult.

In paper [21], it was noted that neural networks make it possible to identify students who need additional support at the early stages of learning, and it was recommended to combine them with adaptive prediction methods.

Adaptive Neuro-Fuzzy Systems (ANFIS) combine the ability of neural networks to learn with the flexibility of fuzzy logic, which allows them to account for uncertainty and change according to student behavior [22, 23]. Thus, study [24] shows that ANFIS allows taking into account both quantitative and qualitative data to predict the academic performance of students in higher education institutions. In paper [25], the author proposed a model that systematically assesses student performance, based on the fuzziness and uncertainty of the data. The model effectively predicts results, showing the advantages of using ANFIS compared to classical machine learning methods.

The authors of work [26] combined the Bayesian approach and ANFIS, which allows for more accurate predictions of student behavior in intelligent learning systems. In paper [27], a neuro-fuzzy model was created to predict academic performance, which takes into account various factors affecting student learning, including individual cognitive characteristics and motivation.

Similar results were shown in other studies [28, 29], where ANFIS allows the integration of fuzzy logic and neural networks, providing flexibility, adaptability, and interpretation of results. In papers [30, 31], the authors demonstrated that such models are effective for personalizing recommendations in e-learning systems, increasing student performance.

In practical e-learning systems, mobile applications and intelligent platforms based on fuzzy logic and neuro-fuzzy systems are used [32–34]. In [35], the authors developed a mobile application for early prediction of student performance using ANFIS, which allows quickly identify students at risk. In articles [36, 37], the authors applied fuzzy logic to cluster students by the level of academic achievement, which increases the adaptability of the system and the personalization of learning.

Other studies [38–40] demonstrate the application of fuzzy logic assessment systems and 3D virtual learning environments that increase predictive accuracy and learning effectiveness. Paper [41] presented an optimized system based on fuzzy logic for predicting students' academic results, which allows integrating the model directly into e-learning platforms.

Thus, the literature review indicates the active development of methods for predicting students' academic performance using machine learning, neural networks, and ANFIS. Neuro-fuzzy systems (ANFIS) show high adaptability and flexibility, combining the ability of neural networks to learn with fuzzy logic. They allow taking into account the uncertainty and fuzziness of students' behavior, which increases the accuracy of forecasts [42, 43]. In addition, ANFIS provide interpretation of results, which is especially important for educators and administrators when making decisions in e-learning systems [44].

Due to these properties, ANFIS is appropriate for integration into modern e-learning platforms, where it is necessary not only to predict student success, but also to adapt the learning process to their individual needs [45]. Applied studies demonstrate that the integration of such models into mobile applications, intelligent learning systems, and 3D virtual environments makes it possible to increase the effectiveness of learning and timely support students at risk [46, 47].

Thus, the use of neuro-fuzzy systems is a reasonable step to improve the accuracy of predicting academic performance and the adaptability of e-learning systems, which makes them promising for the further development of educational technologies.

III. MATHEMATICAL MODEL OF THE E-LEARNING SYSTEM BASED ON FUZZY LOGIC

To build a model of an e-learning system based on fuzzy logic, it is necessary to determine:

1. Input parameters that characterize the learning process.
2. Output parameters that determine the student's success.
3. Fuzzy sets and accessory functions for input and output variables.
4. A system of fuzzy rules of inference, reflecting the laws of the educational process.

Let us consider an e-learning system in which key parameters are recorded for each student:

Factors affecting academic performance are designated as a vector of input characteristics for each student:

$$x = (x_1, x_2, x_3, x_4, x_5, x_6), \quad (1)$$

where x_1 – the level of activity on the platform (the number of completed tasks, tests, lecture views); x_2 – the amount of time spent in the system (in hours); x_3 – average score for completed tasks; x_4 – webinar attendance (percentage); x_5 – participation in discussions on forums and chats; x_6 – results of intermediate tests.

Each variable is normalized to a range [0, 100].

The output variable will be the probability of successful completion of the course: y – probability of successful completion of the course (from 0 to 1) or binary variable $y \in$

{0; 1}, where 0 – the course has not been passed, 1 – successful completion of the course.

This paper proposes a universal model of e-learning system based on fuzzy logic that does not depend on the specific content of the course and can be adapted to various educational disciplines. This is achieved using parameters that reflect the general structure and dynamics of students' educational activity (attendance, academic performance, participation in activities, etc.).

For the purposes of model validation, as well as experiments, data for the last 5 years in the discipline "Distributed and Parallel Computing" were used. The structure of this course included: 15 lectures, 8 laboratory works, 6 practical classes, 2 unit tests, a final control test.

Thus, the developed model demonstrates flexibility and scalability, which allows it to be used to analyze and predict student performance within various courses of the e-learning system. Formula (1) is a generalized description of the structure of any distance course and serves as the basis for the subsequent adaptation of the model to specific conditions.

Now we need to build a fuzzy logical model. To do this, for each input parameter x_i let us define fuzzy sets that correspond to linguistic estimates.

$$x_i \rightarrow \{A_{i1}, A_{i2}, \dots, A_{iL_i}\}, \quad (2)$$

where $A_{ij} \in [0, 100]$ – a fuzzy set corresponding to a linguistic assessment.

The L_i index denotes the number of linguistic terms (fuzzy sets) defined for the input variable x_i . This can be, for example, three terms – "low", "medium", "high". The number may vary depending on the parameter and the selected granularity of the linguistic evaluation scale.

For example, for the variable "activity level", it is possible to define three fuzzy sets:

- Low activity
- Medium activity
- High activity

For each A_{ij} defined membership function:

$$\mu_{A_{ij}}: [0, 100] \rightarrow [0, 1]. \quad (3)$$

Membership functions can be defined as triangular or Gaussian functions:

$$\mu_{Low}(x) = \max\left(0, \frac{50-x}{50}\right) \quad (4)$$

$$\mu_{Medium}(x) = \max\left(0, 1 - \frac{|x-50|}{20}\right) \quad (5)$$

$$\mu_{High}(x) = \max\left(0, \frac{x-50}{50}\right). \quad (6)$$

The membership functions for the remaining input parameters are defined in the same way.

Figure 1 shows a graph of the membership function for the average score parameter.

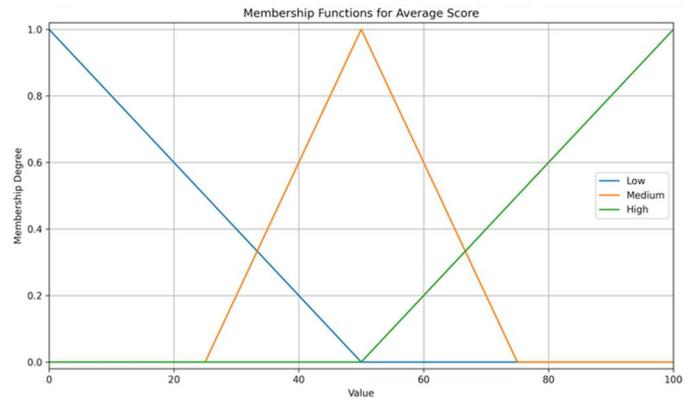


Figure 1. Membership function for the "average score" parameter.

A system of fuzzy inference rules is a set of logical dependencies of the form:

IF (Activity Low) AND (Average Score Low) THEN (Success Rate Low)

IF (Activity Average) AND (Average Score Average) THEN (Success Rate Average)

IF (Activity High) AND (Average Score High) THEN (Success Rate High)

Rules can be formalized as a set of fuzzy production rules:

$$R_k: \text{IF } x_1 \in A_k^1 \text{ AND } x_2 \in A_k^2 \dots x_n \in A_k^n \text{ THEN } y \in B_k, \quad (7)$$

where A_k^i – fuzzy set for the input variable x_i in the k -th rule R_k ; B_k – fuzzy set representing the output value of a result; k – rule index, $k = 1, 2, \dots, M$.

The number of rules M is determined by the number of possible combinations of variable values:

$$M = \prod_{i=1}^6 L_i, \quad (8)$$

where L_i – number of fuzzy sets for a variable x_i .

The activation α_k of each rule R_k is defined using a T-norm operation, that is \min , the operator is used as a logical "AND" to combine all the conditions in the rule:

$$\alpha_k = \min_{i=1 \dots 6} \mu_{A_k^i}(x_i), \quad (9)$$

where α_k shows the degree of truth of the rule R_k for the specified values of the input variables.

In the aggregation phase, the results of all activated rules are combined to produce the resulting fuzzy output. To aggregate the rules, we will use the Mamdani method.

$$\mu_B(y) = \max_{k=1}(\alpha_k * \mu_{B_k}(y)), \quad (10)$$

where $\mu_{B_k}(y)$ – membership function of the output set B_k for each rule.

The max operator combines all partial decisions to form the resulting fuzzy set.

Aggregation combines all partial inferences B_k , weighted by the degree of truth of the corresponding rules, forming the resulting fuzzy set.

The resulting fuzzy set $\mu_B(y)$ needs to be converted to a clear value, in other words, defuzzified, using the Centroid Method:

$$\hat{y} = \frac{\int_0^1 y * \mu_B(y) dy}{\int_0^1 \mu_B(y) dy} \tag{11}$$

The numerator is the weighted sum of the values y considering their membership, and the denominator is the total degree of membership.

The center of gravity method balances the resulting fuzzy set, calculating the expected value to predict a student's success.

The full cycle of data processing in the Mamdani model can be written as:

$$\hat{y} = f(x) = \frac{\sum_{k=1}^M \alpha_k \int_{y_{min}}^{y_{max}} y * \mu_{B_k}(y) dy}{\sum_{k=1}^M \alpha_k \int_{y_{min}}^{y_{max}} \mu_{B_k}(y) dy} \tag{12}$$

where $f(x)$ – function implemented by the Mamdani system; α_k – degree of activation of the k -th rule; $\mu_{B_k}(y)$ – result set membership function.

Thus, the optimization model can be represented as a mapping:

$$f: x \mapsto \hat{y}, f \in \mathcal{F}_{fuzzy} \tag{13}$$

where f – function implemented by a fuzzy output system; x – input variables vector; \hat{y} – prediction of the student's success; \mathcal{F}_{fuzzy} – the space of functions implemented by the fuzzy output system.

The final model of a fuzzy e-learning system is set by a five:

$$FLMS = (X, A, B, R, D) \tag{14}$$

where $X = x_i$ – input variables; A – system of fuzzy sets at the input; B – fuzzy result sets; $R = \{R_k\}$ – rule base; D – defuzzification method.

The presented model of e-learning systems based on fuzzy logic is more resistant to uncertainty and makes it possible to effectively predict learning outcomes.

The Mamdani model allows us to consider the influence of all input parameters and correctly aggregate the results of the rules.

The centroid method provides a stable prediction of the probability of success.

Fuzzy rules based on expert knowledge allow the system to adapt to changes in student behavior, ensuring high accuracy.

IV. THE OPTIMIZATION PROBLEM OF PREDICTING STUDENT SUCCESS

A. FORMULATION

The built mathematical model based on fuzzy logic allows calculating the probability of successful completion of the course.

Let us formulate an optimization problem that will minimize the probability of an error in predicting a student's expulsion.

It is necessary to find such values of the model parameters that minimize the error in predicting the probability of student expulsion.

Let y_i be the real status of the student (0 – the course has not been passed; 1 – has successfully completed the course); \hat{y}_i – the model's predicted probability of successful completion of the course; N – total number of students in the sample.

The goal is to find such a vector of parameters θ , which minimizes the loss function:

$$\theta^* = \arg \min L(\theta) \tag{15}$$

where θ – model parameter vector that includes parameters for membership functions, inference rule weights, and defuzzification parameters.

$$L(\theta) = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2 \tag{16}$$

where $L(\theta)$ – a loss function that reflects the difference between real and predicted values.

Thus, optimization is performed according to the parameters of the fuzzy system: input variable membership functions, output rule weights, defuzzification parameters.

B. SOLUTION

To solve the optimization problem, namely minimizing the loss function $L(\theta)$, one of the following methods can be used.

Gradient methods (e.g., the stochastic gradient descent method, SGD) are effective with a small number of parameters, but can get stuck in local minimum.

Evolutionary algorithms (genetic algorithm, differential evolution) make it possible to find the global minimum but require large computational resources.

Fuzzy neural networks (ANFIS) are adaptive systems that combine the capabilities of fuzzy logic and neural networks to automatically adjust model parameters.

In this paper, we use the ANFIS (Adaptive Neuro-Fuzzy Inference System) method. This method makes it possible to optimize the parameters of a fuzzy system using the backpropagation algorithm and the least squares method.

The ANFIS algorithm consists of the following steps:

1. Initialize fuzzy set parameters. To do this, we need to define the input and output fuzzy variables, centers, and widths of the membership functions. It is possible to specify the initial membership functions (for example, Gaussian or triangular).
2. Training on a training sample:
 - calculating the prediction error at each step;
 - parameter correction using gradient descent and the least squares methods;
 - direct pass: compute the output value \hat{y}_i based on the current parameters;
 - reverse pass: update model parameters based on prediction error.
3. Testing the model on new data.
 - calculating the accuracy of predictions;
 - analysis of model quality on new data.

V. EXPERIMENT AND RESULTS

Real data about students, including their activity, grades, and attendance, was used to test the model.

Table 1 shows a part of the dataset that was used for the experiment.

Table 1. Example of input data

| Activity | Time Spent | Average Score | Attendance | Discussion | Test Results |
|----------|------------|---------------|------------|------------|--------------|
| 51 | 56 | 62 | 95 | 7 | 62 |
| 92 | 16 | 85 | 63 | 25 | 21 |
| 71 | 89 | 87 | 88 | 29 | 57 |
| 60 | 43 | 71 | 9 | 6 | 56 |
| 20 | 24 | 50 | 20 | 66 | 76 |
| 82 | 16 | 63 | 45 | 64 | 3 |

Each of the six traits is assigned a weight:

$$\begin{aligned} score = & 0.1 * x_1 + 0.1 * x_2 + 0.3 * x_3 + \\ & + 0.2 * x_4 + 0.1 * x_5 + 0.2 * x_6 \end{aligned} \quad (17)$$

More weight is given to the grade, attendance and test results are key indicators.

Less weight is given to activity, time in the system and participation in discussions.

The weights in formula (17) were determined based on statistical analysis of data collected over the past five years across several disciplines implemented in e-learning system. In particular, aggregated data from the courses “Distributed and Parallel Computing” and “Architecture of Computing Systems” were used.

The analysis included the frequency of each trait in students who successfully completed the course; assessment of their impact on the likelihood of expulsion; the relative importance of the components (tests, labs, lectures, etc.) in terms of correlation with the final result.

Thus, the weights reflect the real statistical impact of factors on academic performance and can be used as initial values for further optimization when training the model, for example, using the **neuro-fuzzy** approach or gradient hyperparameter search methods.

Let us conduct an experiment to train an ANFIS model and analyze the results.

At the same time, the real data is unstable. To simulate this, let us add random noise to the final result:

$$score_{noisy} = score + \varepsilon, \varepsilon \sim \mathcal{N}(0,5). \quad (18)$$

This reflects the instability of the student’s behavior, motivation and external factors.

The result obtained is compared with the threshold (traditionally 60 points) to decide whether the student will be expelled or will successfully pass the disciplinary assignment:

$$\hat{y} = \begin{cases} 1, & \text{if } score_{noisy} \geq 60 \\ 0, & \text{otherwise} \end{cases}. \quad (19)$$

This model accepts multidimensional inputs, weighs the significance of factors, and allows for uncertainty, which approximates the classical fuzzy logical system, where weights can be interpreted as the result of fuzzy rules and conclusions.

The optimized model was tested on a sample of students. The assessment of accuracy showed that the use of fuzzy logic made it possible to achieve high accuracy in predicting the expulsion of students.

Figure 2 shows the ROC curve for an experiment with 1000 students.

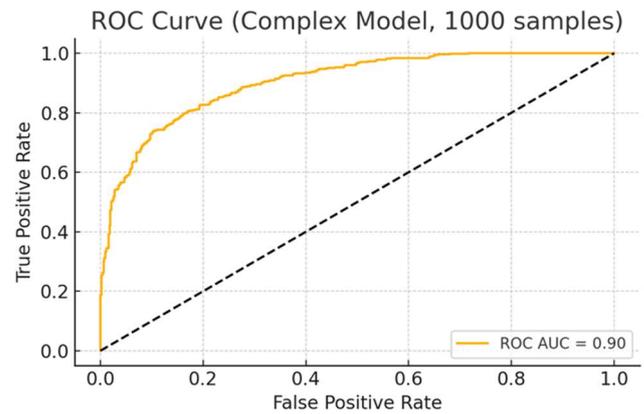


Figure 2. ROC curve for an experiment with 1000 students.

The value $AUC = 0.90$ in tests per 1000 students indicates that the model correctly distinguishes positive samples from negative samples 90% of the time. The entire curve lies above the diagonal, which confirms the effectiveness of the algorithm.

The ROC curve of the model rises steadily to the upper left corner, which confirms a good ratio between TPR and FPR at different thresholds. At a low level, FPR is already achieved $TPR > 0.7$ – this is good for tasks where it is important to classify as much as possible correctly.

The ROC curve shows that the model has a balanced ratio of completeness and accuracy and can effectively predict the probability of student expulsion, minimizing both false positive and false negative errors.

A high AUC value indicates that the model can accurately distinguish between successful students and those at risk of expulsion.

The ROC AUC remains high, indicating a good ability to distinguish between classes.

Table 2. Classification report of testing the model per 1000 students

| | precision | recall | f1-score | support |
|---------------------|-----------|----------|----------|---------|
| Class 1 | 0.814393 | 0.83495 | 0.82454 | 515.0 |
| Class 2 | 0.81992 | 0.79794 | 0.80878 | 485.0 |
| accuracy | 0.817 | 0.817 | 0.817 | 0.817 |
| macro avg | 0.81715 | 0.816442 | 0.81666 | 1000.0 |
| weighted avg | 0.81707 | 0.817 | 0.81689 | 1000.0 |

Both metrics are almost identical, suggesting that classes are balanced (Class 1: 515 students, Class 2: 485 students) and that the model is not biased in favor of one of them.

Figure 3 shows a confusion matrix for an experiment with 1000 students

Confusion Matrix (Complex Model, 1000 samples)

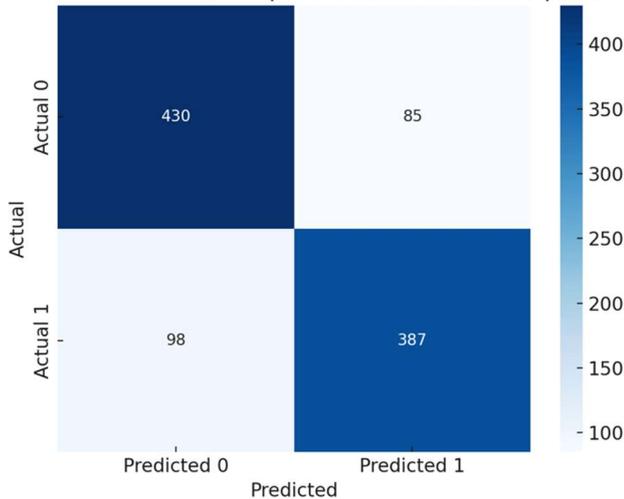


Figure 3. Confusion matrix for an experiment with 1000 students.

The confusion matrix when testing the model on 1000 students showed the following results:

- 430 correctly predicted expulsions (True Negatives)
- 387 correctly predicted successful course completions (True Positives)
- 85 the model incorrectly predicted success (False Positives)
- 98 the model did not recognize a successful student (False Negatives)

The model remains balanced and stable, close to the behavior of a real fuzzy system.

The accuracy of the model is calculated as:

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN} = \frac{387 + 430}{387 + 430 + 85 + 98} = \frac{817}{1000} = 81.7\%. \quad (20)$$

The final accuracy rate is 81.7%, which shows that the model reliably classifies students and is suitable for predicting expulsion under uncertainty.

VI. CONCLUSION

As part of this work, a mathematical model of the e-learning system was developed and tested based on fuzzy logic presented by the five $FLMS = (X, A, B, R, D)$. The model successfully formalizes uncertainty in learning processes and makes it possible to predict the probability of successful completion of the course for each student.

The results of testing the model on 1000 examples showed high classification accuracy: the final accuracy was 81.7%, the value of $AUC = 0.90$ confirmed the high ability of the model to distinguish students who are prone to expulsion from successful ones. The model provides a balanced classification, with close precision and recall values for both groups (successful and unsuccessful students), which indicates that there is no bias and tolerance for class skew.

The confusion matrix showed that out of 1000 students, the model correctly identified 430 cases of expulsion (True Negatives), 387 successful courser completion (True Positives), while the number of errors remained at an acceptable level (85 false positives and 98 false negative predictions).

The practical value of the model lies in its application in educational platforms for:

- early diagnosis of the risk group among students;
- adapting educational content and teaching methods to the needs of students;
- optimizing the learning process by concentrating resources on students who need additional support.

In addition, the model is adaptable to various conditions – it can be reconfigured to the specifics of a particular online platform or training course using a flexible base of rules and methods of defuzzification.

Thus, the developed fuzzy model demonstrates high efficiency and is a promising tool in the tasks of data mining in education.

VII. FURTHER RESEARCH

Further research will focus on extending and improving the proposed methodology in several complementary directions. One promising area is the development of hybrid models that integrate fuzzy inference systems with deep neural networks. While the current approach demonstrates the effectiveness of neuro-fuzzy modeling for predicting students' learning outcomes, deep learning architectures may enhance the model's ability to capture complex, nonlinear relationships in large-scale educational data. Combining interpretability of fuzzy rules with the representational power of deep neural networks could lead to more robust and scalable predictive models.

Another important direction for future work is the inclusion of additional psychological and social factors influencing students' academic performance. Variables such as motivation, self-regulation, stress levels, peer interaction, and learning environment characteristics are known to significantly affect learning outcomes but are not fully represented in standard e-learning datasets. Incorporating such factors into the model may improve forecasting accuracy and provide a more holistic assessment of students' learning ability.

Further research will also address the optimization of computational complexity to enable real-time operation of the model. As e-learning systems increasingly require immediate feedback and adaptive decision-making, reducing training and inference time becomes critical. This may involve simplifying the rule base, applying feature selection techniques, or exploring more efficient training algorithms without compromising prediction accuracy.

The developed methodology has strong potential for automated analysis of students' learning abilities and the generation of personalized educational trajectories. Future studies will focus on transforming model outputs into actionable recommendations, such as adaptive content sequencing, individualized difficulty levels, and targeted learning interventions. This will strengthen the practical value of the approach for intelligent tutoring systems.

In addition, it is planned to integrate the proposed model into a real e-learning platform for large-scale experimental validation. Conducting practical experiments in educational institutions of Ukraine will allow evaluation of the model under real learning conditions, comparison with existing assessment methods, and analysis of its impact on students' academic performance and engagement.

Finally, further work will aim to optimize the fuzzy rule structure and training procedures to improve prediction accuracy to 85% or higher. This includes refining membership functions, reducing redundant rules, and exploring advanced learning

strategies such as evolutionary optimization and ensemble approaches. These improvements are expected to enhance both the accuracy and interpretability of the model, making it more suitable for practical educational applications.

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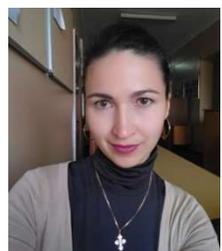
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