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NUMERICAL MODELING OF MOMENT METHOD FOR ANTENNA SIMULATION

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Abstract: *Paper includes accuracy problems of numerical computing of wire antennas characteristics, specially resistively loaded dipoles - antennas with varying inner impedance. One may find here the modification of Pocklington's equation (numerical computation of resistive dipole) to the form, which allows us to decrease the number of dipole segmentation elements.*

Keywords: *computing, numerical methods, antenna modelling, current distribution, method of moment.*

1 INTRODUCTION

Method of moment (MoM) is numerical method for wire antenna analysis which is the most frequently used to find the solution of internal and external antenna task of given antenna type. If the MoM is based upon Hallen's integral equation (without source function), the task solution is relatively simple. In this reason it may be used for current distribution analysis. But if the antenna is made from resistive material or its internal resistance is varying with position on antenna arms, the problem of numerical computing becomes more complicated. Hallen's integral equation can not be used. It must be replaced by Pocklington's one, on the right side of which not only source function is present, but also the multiplication function of internal wire impedance and the unknown current. Typical example of this antenna type is resistively loaded dipole with travelling wave. It is often used as electric field sensor in electromagnetic compatibility area for immunity testing [1], [2] and [3]. As it could be found in [3], the non perfection antenna simulation was very time and (CPU time) consuming. For this reasons it was desired to find out such way of Pocklington's equation solution, which could remove this shortages. As written in following text the solution was found by the connection of numerical and analytical integration of separate parts of the equation and by utilising the properties of impedance matrix.

2 PROBLEM FORMULATION

On the base of [1], in which the resistive dipole is described in more details, the figure 1 was prepared. The antenna consists of a cylinder of length *2h* and its wire diameter is *2a*. It has an internal impedance $Z^i(z)$, due to continuous complex impedance loading, and carries an axial current $I_z(z)$, which is assumed to be uniformly around the periphery of the cylinder since the radius is much less than the wavelength, λ. According to [2] one can suppose, that the current is flowing only in *z* axis direction and it is equal to zero in the point $z=h$. The dipole is made from non perfection conductors material, so that its internal impedance satisfies the equation (3). The dipole is fed in the point given by $z=0$ by voltage U_0. On the base of this prepositions the axial component of vector potential must satisfy one-dimensional wave equation:

$$\left(\frac{\partial^2}{\partial z^2}+k^2\right)A_z(z)=\frac{jk^2}{\omega}\left[Z^i(z)I_z(z)-U_0\delta(z)\right] \quad (1)$$

where $A_z(z)$ is vector potential

$$A_z(z) = \frac{\mu}{4\pi}\int_{-h}^{h} I_z(z')\frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}} \quad (2)$$

while

2h is physical length of antenna [m]
k is wave number [rad . m^{-1}]
ω is angle frequency [rad . s^{-1}]
$\mu 0$ is vacuum permeability [H . m^{-1}]
$\delta(z)$ is Dirac function [1]

The internal dipole arm wire impedance $Z^i(z)$ in (1) is given by [1]

$$Z^i(z) = \frac{\Psi}{h - |z|} \quad (3)$$

where Ψ is the constant dependent on dipole dimensions and conductivity of dipole materials.

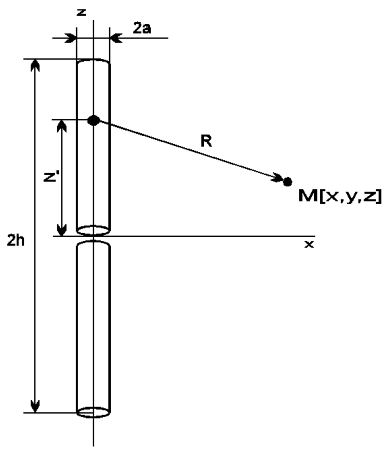

Fig. 1 - The geometry of resistive loaded dipole

3 STANDARD NUMERICAL MODELLING

Method of moment, as was already written, is advantageous for integral equation solution. The basis of this method is in division of integral equation into the system of algebraic equations, which are simply soluble. In praxis it means to divide the dipole into *N* segments and to express the current flowing in each segment. After some modification the equation (1) must obtain the following form

$$\sum \mathbf{I}_n \cdot \mathbf{Z}_{nm} = \mathbf{U}_m \quad (4)$$

where I_n is the vector of unknown currents, U_m is the vector of dipole feeding voltages, which contains only voltage U_0 at the point of dipole feeding, i. e. for N/2-th segment and zeros for other segments. Z_{nm} is impedance matrix calculated by following way: by means of figure 1 the position of vector oriented from the point N on the dipole to the point of observation M is defined. Its length is equal to

$$R_{zz'} = \sqrt{(z-z')^2 + a^2} \quad (5)$$

The derivation in (1) must be put under the integral and then the integrand has the form

$$\Xi_{yy'} = \frac{d}{dz}G_{zz'} + k^2 G_{zz'} \quad (6)$$

where

$$G_{zz'} = \frac{e^{-jkR_{zz'}}}{4\pi R_{zz'}} \quad (7)$$

On the basis of preceding equation the impedance Z_{nm} is calculated as:

$$Z_{nm} = \frac{1}{j2\pi f\varepsilon_0}\int_{z_{n-1/2}}^{z_{n+1/2}} \Xi_{z_m z'}\,dz + Z^i(z_m)\left(z_{m+1/2} - z_{m-1/2}\right) \quad (8)$$

Current distribution calculation according to (4) using (8) is a time consuming one, while it must perform *n.m* numerical derivations and integration. Even the suitable simulation results will come only after increasing the number of segments to 600 or more (So the matrix of *600 . 600* complex numbers will be obtained, which requires a corresponding hardware aids) [3].

4 ADVANCED NUMERICAL MODELLING

Since the solution of equation (4) and (8) is very difficult to realise on obvious hardware means (PC computer), such a way of the equation (1) solution has to be found, that reaches satisfactory results with minimum number of segments and numerical derivations and integration. One way how to solve this problem is the division of the equation (4) into two separate parts. This seems to be a suitable solution. One part will contain only the impedance dependent upon the current distribution (the first part of the equation (8)) and the second only those impedance, which depend on the shape and the material, from which the dipole is made (the second part of the equation (8))

$$\mathbf{Z}_{nm} = \mathbf{Z}'_{nm} + \mathbf{Z}^{i}_{nm} \tag{9}$$

Utilising the features of moment method applied for Pocklington's equation for the matrix Z'_{nm} and utilising the fact, that for impedance calculation the length of the position vector is needed and not its orientation, the equation (10) can be written. For example the length of vector $R_{zz'}$ from point 1 to point 1 is the same as the length from any point to the same point, so all diagonal elements are the same. Also in our case the length of vector connecting neighbouring segments are the same. Based on previous statement, the following equation can be presented:

$$\mathbf{Z}'_{nm} = \begin{pmatrix} Z'_1 & Z'_2 & Z'_3 & Z'_4 & Z'_5 & \cdots \\ Z'_2 & Z'_1 & Z'_2 & Z'_3 & Z'_4 & \cdots \\ Z'_3 & Z'_2 & Z'_1 & Z'_2 & Z'_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{10}$$

The fact, that it is not necessary to solve *m . n* numerical integration and derivations, follows from equation (10). But it is needed only to calculate the first matrix row and all remaining rows will are only the combinations of the elements of the first row. To increase the precision of internal impedance estimation of each dipole segment, it is required to modify the calculation of Z^{i}_{mn} according to following formula

$$Z^{i}_{nm} = \int_{m-1/2}^{m+1/2} \frac{\Psi}{h-|z|} dz \tag{11}$$

From (11) one can obtain:

$$Z^{i}_{nm} = \begin{cases} \Psi\left(\ln\left(z_{m-1/2} - h\right) - \ln\left(z_{m+1/2} + h\right)\right) & z > 0 \\ \Psi\left(\ln\left(h + z_{m-1/2}\right) - \ln\left(h + z_{m+1/2}\right)\right) & z < 0 \end{cases} \tag{12}$$

On the basis of last two equations one can create the matrix of internal impedance

$$\mathbf{Z}^{i}_{nm} = \begin{pmatrix} Z_1 & Z_1 & Z_1 & \cdots \\ Z_2 & Z_2 & Z_2 & \cdots \\ Z_3 & Z_3 & Z_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{13}$$

When summarised equations (10) and (13), the classic method of moments is obtained. However, in this case it is necessary to solve only the system of linear equations according to (4).

5 RESULT OF THE BOTH METHODS

On the basis of the analysis made in chapters 2 and 3 the programs solving the given problem were created in the *Mathematica* environment. This software tool is designed to solve mathematical problems, which gives to the operator an user friendly access to effective algorithm writing of particular equations required for calculations of current distribution and other parameters. The task can be divided into several functions, which solve particular problems and by means of them one can calculate the problem solution. This feature allows us to change input parameters of simulation (dipole dimensions, material parameters, number of segments, required characteristics) very simply. The effective antenna height can be also calculated according to formula

$$h_e(f) = \frac{1}{I_z(0)} \sum_{m=1}^{M} I_z(z_m)\left(z_{m+1/2} - z_{m-1/2}\right) \tag{14}$$

To check the behaviour of proposed method the current distribution along resistive dipole with dimensions of 2h=30 cm and a=1 mm has been calculated. Numerical calculation was made for dipole divided into 121 segments. Then the real part of effective antenna height was calculated. The resulting frequency dependence was compared to analytical solution given in literature [3]. The comparison can be seen in figure 2, which shows very good agreement of both results.

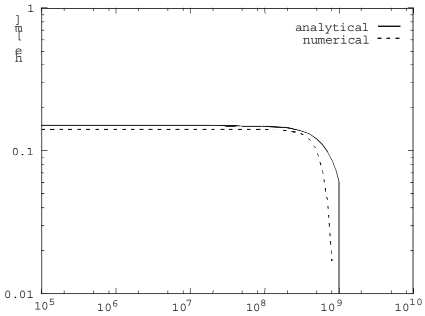

Fig. 2 - The effective dipole height - real component

6 CONCLUSION

The paper describes new modification of resistive dipoles solution by method of moments. Proposed method can be applied also for standard wire dipoles, which are made from real metallic material with finite conductivity. The next contribution of the paper is the fact, that the given method allows us to use cheap computer aids for solving complicated antenna problems in stead of more powerful computers. This needs only the modification of mathematical formulation of solved problem. Time required for calculations according to proposed method is less than 0.1 of the previous one described in [3]. This gives us an opportunity to use classical PC for antenna problem solving.

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